Twisted spinors on Schwarzschild and Reissner-Nordström black holes

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Abstract

We describe twisted configurations of spinor field on the Schwarzschild and Reissner-Nordström black holes that arise due to existence of the twisted spinor bundles over the standard black hole topology $\mathbb{R}^2 \times \mathbb{S}^2$. From a physical point of view the appearance of spinor twisted configurations is linked with the natural presence of Dirac monopoles that play the role of connections in the complex line bundles corresponding to the twisted spinor bundles. Possible application to the Hawking radiation is also outlined.

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1 Introductory remarks

Recently it has appeared an interest in studying topologically inequivalent configurations (TICs) of various fields on the 4D black holes [1–4] since TICs might give marked additional contributions to the quantum effects in the 4D black hole physics, for instance, such as the Hawking radiation [2] and also might help to solve the problem of statistical substantiation of the black hole entropy [3]. So far, however, only TICs of complex scalar field have been studied more or less on the Schwarzschild (SW), Reissner-Nordström (RN) and Kerr black holes. The next physically important case is the one of spinor fields. The present paper will be, therefore, devoted just to a description of twisted TICs of spinor field in the form convenient to physical applications within the framework of the SW and RN black hole geometry.

As was discussed in Refs. [1–4], TICs exist owing to high nontriviality of the standard topology of the 4D black hole spacetimes which is of the $\mathbb{R}^2 \times \mathbb{S}^2$ -form. High nontriviality of the given topology consists in the fact that over it there exist a huge (countable) number of nontrivial real and complex vector

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bundles of any rank $N > 1$ (for complex ones for $N = 1$ too). In particular, TICs of complex scalar field on the 4D black holes are conditioned by the availability of countable number of complex line bundles over the $\mathbb{R}^2 \times \mathbb{S}^2$ -topology underlying the 4D black hole physics. In turn, TICs of spinor field can be tied with the twisted spinor bundles on the given topology.

We write down the black hole metrics under discussion (using the ordinary set of local coordinates t, r, ϑ, φ) in the form

$$ds^2 = g_{\mu\nu} dx^\mu \otimes dx^\nu \equiv adt^2 - a^{-1}dr^2 - r^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2) \tag{1}$$

with $a = 1 - 2M/r + \alpha^2 M^2/r^2$, $\alpha = Q/M$, where M, Q are, respectively, a black hole mass and a charge. Besides we have $|g| = |\det(g_{\mu\nu})| = (r^2 \sin\vartheta)^2$, $r_\pm = M(1 \pm \sqrt{1-\alpha^2})$ with $0 \le \alpha \le 1$, so $r_+ \le r < \infty$, $0 \le \vartheta < \pi$, $0 \le \varphi < 2\pi$.

Throughout the paper we employ the system of units with $\hbar = c = G = 1$, unless explicitly stated otherwise. Finally, we shall denote $L_2(F)$ the set of the modulo square integrable complex functions on any manifold F furnished with an integration measure.

2 Description of TICs

Mathematical grounds for the existence of spinor field TICs on black holes lie in the fact that over the standard black hole topology $\mathbb{R}^2 \times \mathbb{S}^2$ there exists only one Spin-structure [conforming to the group Spin(1,3)= $\mathrm{SL}(2, \mathbb{C})$]. Referring for the exact definition of Spin-structure to Refs.[5,6], we here only note that the number of inequivalent Spin-structures for manifold M is equal to the one of elements in $H^1(M, \mathbb{Z}_2)$, the first cohomology group of M with coefficients in $\mathbb{Z}_2$. In our case $H^1(\mathbb{R}^2 \times \mathbb{S}^2, \mathbb{Z}_2) = H^1(\mathbb{S}^2, \mathbb{Z}_2)$ which is equal to 0 and thus there exists the only (trivial) Spin-structure.

On the other hand, the nonisomorphic complex line bundles over the $\mathbb{R}^2 \times \mathbb{S}^2$ -topology are classified by the elements in $H^2(M, \mathbb{Z})$, the second cohomology group of M with coefficients in $\mathbb{Z}$ [1], and in our case this group is equal to $H^2(\mathbb{S}^2, \mathbb{Z}) = \mathbb{Z}$ and, consequently, the number of complex line bundles is countable. As a result, each complex line bundle can be characterized by an integer $n \in \mathbb{Z}$ which in what follows will be called its Chern number.

Under this situation, if denoting $S(M)$ the only standard spinor bundle over $M = \mathbb{R}^2 \times \mathbb{S}^2$ and ξ the complex line bundle with Chern number n, we can construct tensorial product $S(M) \otimes \xi$. As is known [7], over any noncompact spacetime the bundle $S(M)$ is trivial and, accordingly, the Chern number of 4-dimensional vector bundle $S(M) \otimes \xi$ is equal to n as well. Under the circumstances we obtain the *twisted Dirac operator* $\mathcal{D} : S(M) \otimes \xi \to S(M) \otimes \xi$,

so the wave equation for corresponding spinors Ψ (with a mass μ_0) as sections of the bundle $S(M)\otimes\xi$ may look as follows

$$\mathcal{D}\Psi=\mu_0\Psi, \tag{2}$$

and we can call (standard) spinors corresponding to $n=0$ (trivial complex line bundle ξ) *untwisted* while the rest of the spinors with $n\neq 0$ should be referred to as *twisted*.

From general considerations [5,6,8] the explicit form of the operator $\mathcal{D}$ in local coordinates x^μ on a $2k$-dimensional (pseudo)riemannian manifold can be written as follows

$$\mathcal{D}=i\nabla_\mu\equiv i\gamma^c E_c^\mu(\partial_\mu-\frac{1}{2}\omega_{\mu ab}\gamma^a\gamma^b-ieA_\mu),\ a<b, \tag{3}$$

where $A=A_\mu dx^\mu$ is a connection in the bundle ξ and the forms $\omega_{ab}=\omega_{\mu ab}dx^\mu$ obey the Cartan structure equations $de^a=\omega^a_{\ b}\wedge e^b$ with exterior derivative d, while the orthonormal basis $e^a=e^a_\mu dx^\mu$ in cotangent bundle and dual basis $E_a=E^\mu_a\partial_\mu$ in tangent bundle are connected by the relations $e^a(E_b)=\delta^a_b$. At last, matrices γ^a represent the Clifford algebra of the corresponding quadratic form in $\mathbb{C}^{2^k}$. Below we shall deal only with 2D euclidean case (quadratic form $Q_2=x_0^2+x_1^2$) or with 4D lorentzian case (quadratic form $Q_{1,3}=x_0^2-x_1^2-x_2^2-x_3^2$). For the latter case we take the following choice for γ^a

$$\gamma^0=\begin{pmatrix}1&0\\0&-1\end{pmatrix},\gamma^b=\begin{pmatrix}0&\sigma_b\\-\sigma_b&0\end{pmatrix},b=1,2,3\,, \tag{4}$$

where σ_b denote the ordinary Pauli matrices

$$\sigma_1=\begin{pmatrix}0&1\\1&0\end{pmatrix},\sigma_2=\begin{pmatrix}0&-i\\i&0\end{pmatrix},\sigma_3=\begin{pmatrix}1&0\\0&-1\end{pmatrix}\,. \tag{5}$$

It should be noted that further in lorentzian case, Greek indices $\mu,\nu,...$ will be raised and lowered with $g_{\mu\nu}$ of (1) or its inverse $g^{\mu\nu}$ and Latin indices $a,b,...$ will be raised and lowered by $\eta_{ab}=\eta^{ab}$= diag(1,-1,-1,-1), so that $e^a_\mu e^b_\nu g^{\mu\nu}=\eta^{ab}$, $E^\mu_a E^\nu_b g_{\mu\nu}=\eta_{ab}$ and so on.

Using the fact that all the mentioned bundles $S(M)\otimes\xi$ can be trivialized over the chart of local coordinates (t,r,ϑ,φ) covering almost the whole manifold $\mathbb{R}^2\times\mathbb{S}^2$, we can concretize the wave equation (2) on the given chart for TIC Ψ with the Chern number $n\in\mathbb{Z}$ in the case of metric (1). Namely, we can put $e^0=\sqrt{a}dt$, $e^1=dr/\sqrt{a}$, $e^2=rd\vartheta$, $e^3=r\sin\vartheta d\varphi$ and, accordingly, $E_0=\partial_t/\sqrt{a}$, $E_1=\sqrt{a}\partial_r$, $E_2=\partial_\vartheta/r$, $E_3=\partial_\varphi/(r\sin\vartheta)$. This entails

$$\omega_{01}=-\frac{1}{2}\frac{da}{dr}dt,\omega_{12}=-\sqrt{a}d\vartheta,\omega_{13}=-\sqrt{a}\sin\vartheta d\varphi,\omega_{23}=-\cos\vartheta d\varphi. \tag{6}$$

As for the connection A_μ in bundle ξ then the suitable one was found in Refs.[1] and is

$$A = A_\mu dx^\mu = -\frac{Q}{r}dt - \frac{n}{e}\cos\vartheta d\varphi\,. \tag{7}$$

Under the circumstances, as was shown in Refs. [1], integrating $F = dA$ over the surface $t = const$, $r = const$ with topology $\mathbb{S}^2$ gives rise to the Dirac charge quantization condition

$$\int\limits_{S^2} F = 4\pi\frac{n}{e} = 4\pi q \tag{8}$$

with magnetic charge q, so we can identify the coupling constant e with electric charge. Besides, the Maxwell equations $dF = 0$, $d*F = 0$ are fulfilled [1] with the exterior differential $d = \partial_t dt + \partial_r dr + \partial_\vartheta d\vartheta + \partial_\varphi d\varphi$ in coordinates t, r, ϑ, φ, where $*$ means the Hodge dual form. We come to the same conlusion that in the case of TICs of complex scalar field [1–4]: the Dirac magnetic $U(1)$-monopoles naturally live on the black holes as connections in complex line bundles and hence physically the appearance of TICs for spinor field should be obliged to the natural presence of Dirac monopoles on black hole and due to the interaction with them the given field splits into TICs. Also it should be emphasized that the total (internal) magnetic charge Q_m of black hole which should be considered as the one summed up over all the monopoles remains equal to zero because

$$Q_m = \frac{1}{e}\sum_{n\in\mathbb{Z}} n = 0\,. \tag{9}$$

Returning to the Eq. (2), we can see that with taking into account all the above it takes the form

$$\left[i\gamma^0\frac{1}{\sqrt{a}}\left(\partial_t - \frac{1}{2}\omega_{t01}\gamma^0\gamma^1 + \frac{ieQ}{r}\right) + i\gamma^1\sqrt{a}\partial_r + i\gamma^2\frac{1}{r}\left(\partial_\vartheta - \frac{1}{2}\omega_{\vartheta 12}\gamma^1\gamma^2\right) + \right.$$
$$\left. i\gamma^3\frac{1}{r\sin\vartheta}\left(\partial_\varphi - \frac{1}{2}\omega_{\varphi 13}\gamma^1\gamma^3 - \frac{1}{2}\omega_{\varphi 23}\gamma^2\gamma^3 + in\cos\vartheta\right)\right]\Psi = \mu_0\Psi\,. \tag{10}$$

After a simple matrix algebra computation with using (4), (6) and the ansatz $\Psi = e^{i\omega t}r^{-1}\begin{pmatrix} F_1(r)\Phi(\vartheta,\varphi) \\ F_2(r)\sigma_1\Phi(\vartheta,\varphi)\end{pmatrix}$ with a 2D spinor $\Phi = \begin{pmatrix}\Phi_1 \\ \Phi_2\end{pmatrix}$, we can from (10) obtain the system

$$\sqrt{a}\partial_r F_1 + \left(\frac{1}{2}\frac{d\sqrt{a}}{dr} + \frac{\lambda}{r}\right)F_1 = i(\mu_0 - c)F_2,$$
$$\sqrt{a}\partial_r F_2 + \left(\frac{1}{2}\frac{d\sqrt{a}}{dr} - \frac{\lambda}{r}\right)F_2 = -i(\mu_0 + c)F_1 \tag{11}$$

with an eigenvalue λ for the eigenspinor Φ of the operator $D_n = -i\sigma_1[i\sigma_2\partial_\vartheta + i\sigma_3\frac{1}{\sin\vartheta}(\partial_\varphi - \frac{1}{2}\sigma_2\sigma_3\cos\vartheta + in\cos\vartheta)]$, so that $\sigma_1 D_n = -D_n\sigma_1$, while $c = \frac{1}{\sqrt{a}}(\omega + \frac{eQ}{r})$. We need, therefore, explore the equation $D_n\Phi = \lambda\Phi$.

3 Spectrum of the operator D_n

As is not complicated to see, the operator D_n has the form (3) with $\gamma^0 = -i\sigma_1\sigma_2$, $\gamma^1 = -i\sigma_1\sigma_3$, $e^0 = d\vartheta$, $e^1 = \sin\vartheta d\varphi$, $\omega_{01} = \cos\vartheta d\varphi$, $A_\mu dx^\mu = -\frac{n}{e}\cos\vartheta d\varphi$, i. e., it corresponds to the abovementioned quadratic form Q_2 and this is just twisted (euclidean) Dirac operator on the unit sphere and the conforming complex line bundle is the restriction of bundle ξ on the unit sphere. Again simple matrix algebra computation results in $D_n = \begin{pmatrix} D_{1n} & D_{2n} \\ -D_{2n} & -D_{1n} \end{pmatrix}$ with $D_{1n} = i(\partial_\vartheta + \frac{1}{2}\cot\vartheta)$, $D_{2n} = -\frac{1}{\sin\vartheta}(\partial_\varphi + in\cot\vartheta)$. Then it is easy to see that the equation $D_n\Phi = \lambda\Phi$ can be transformed into the one

$$\begin{pmatrix} 0 & D_n^- \\ D_n^+ & 0 \end{pmatrix}\Phi_0 = \lambda\Phi_0, \Phi_0 = \begin{pmatrix} \Phi_+ \\ \Phi_- \end{pmatrix}, \tag{12}$$

where $D_n^\pm = D_{1n} \pm D_{2n} = i[\partial_\vartheta + (\frac{1}{2} \mp n)\cot\vartheta] \mp \frac{1}{\sin\vartheta}\partial_\varphi$, $\Phi_\pm = \Phi_1 \pm \Phi_2$. From here it follows that $D_n^- D_n^+ \Phi_+ = \lambda^2\Phi_+$, $D_n^+ D_n^- \Phi_- = \lambda^2\Phi_-$ or, with employing the ansatz $\Phi_\pm = P_\pm(\vartheta)e^{-im'\varphi}$, in explicit form

$$\left(\partial_\vartheta^2 + \cot\vartheta\partial_\vartheta - \frac{m'^2 + (n \mp 1/2)^2 - 2m'(n \mp 1/2)\cos\vartheta}{\sin^2\vartheta}\right)P_\pm(\vartheta) =$$

$$\left(\frac{1}{4} - n^2 - \lambda^2\right)P_\pm(\vartheta)\,. \tag{13}$$

It is known [9] that differential operator

$$\Delta_\vartheta = \partial_\vartheta^2 + \cot\vartheta\partial_\vartheta - \frac{m'^2 + n'^2 - 2m'n'\cos\vartheta}{\sin^2\vartheta} \tag{14}$$

has eigenfunctions in the interval $0 \le \vartheta \le \pi$, which are finite at $\vartheta = 0, \pi$, only for eigenvalues $-k(k+1)$, where k is positive integer or half-integer simultaneously with m', n' while the multiplicity of such an eigenvalue is equal to $2k+1$. In our case of Eq.(13) we have that $n' = n \pm 1/2$ is half-integer because the Chern number $n \in \mathbb{Z}$. As a result, we should put $m' = m - 1/2$ with an integer m and then $|m'| \le k = l + 1/2$ with a positive integer l and, accordingly, $1/4 - n^2 - \lambda^2 = -k(k+1)$ which entails (denoting $\lambda = \sqrt{(l+1)^2 - n^2}$) that spectrum of D_n consists of the numbers $\pm\lambda$ with multiplicity $2k+1 = 2(l+1)$ of each one. Besides, it is clear that under this $-l \le m \le l+1, l \ge |n|$. This just reflects the fact that from general considerations [5,6,8] the spectrum of twisted euclidean Dirac operator on even-diemensional manifold is symmetric with respect the origin. At $n = 0$ we get the spectrum of just Dirac operator on $\mathbb{S}^2$, i. e., $\pm\lambda = \pm(l+1) \in \mathbb{Z}\backslash\{0\}$ which may be obtained by purely algebraic methods (see, e. g. Refs.[10]). The corresponding eigenfunctions of the above Δ_ϑ-operator can be chosen by miscellaneous ways, for instance, as follows (see,

e. g., Ref. [9])

$$P^k_{m'n'}(\cos\vartheta)=i^{-m'-n'}\sqrt{\frac{(k-m')!(k-n')!}{(k+m')!(k+n')!}}\left(\frac{1+\cos\vartheta}{1-\cos\vartheta}\right)^{\frac{m'+n'}{2}}\times$$

$$\times\sum_{j=\max(m',n')}^{k}\frac{(k+j)!i^{2j}}{(k-j)!(j-m')!(j-n')!}\left(\frac{1-\cos\vartheta}{2}\right)^{j} \tag{15}$$

with the orthogonality relation at n' fixed

$$\int\limits_0^\pi P^{*k}_{m'n'}(\cos\vartheta)P^{k'}_{m''n'}(\cos\vartheta)\sin\vartheta d\vartheta=\frac{2}{2k+1}\delta_{kk'}\delta_{m'm''}\,, \tag{16}$$

where (*) signifies complex conjugation. As a consequence, we come to the conclusion that spinor Φ_0 of (12) can be chosen in the form $\Phi_0=\begin{pmatrix}C_1P^k_{m'n-1/2}\\ C_2P^k_{m'n+1/2}\end{pmatrix}e^{-im'\varphi}$ with some constants $C_{1,2}$. Now we can employ the relations [9]

$$-\partial_\vartheta P^k_{m'n'}\pm\left(n'\cot\vartheta-\frac{m'}{\sin\vartheta}\right)P^k_{m'n'}=-i\sqrt{k(k+1)-n'(n'\pm1)}P^k_{m'n'\pm1} \tag{17}$$

holding true for functions $P^k_{m'n'}$ to establish that $C_1=C_2=C$ corresponds to eigenvalue λ while $C_1=-C_2=C$ conforms to $-\lambda$. Thus, the eigenspinors $\Phi=\begin{pmatrix}\Phi_1\\ \Phi_2\end{pmatrix}$ of the operator D_n can be written as follows

$$\Phi_{\pm\lambda}=\frac{C}{2}\begin{pmatrix}P^k_{m'n-1/2}\pm P^k_{m'n+1/2}\\ P^k_{m'n-1/2}\mp P^k_{m'n+1/2}\end{pmatrix}e^{-im'\varphi}\,, \tag{18}$$

where the coefficient C may be defined from the normalization condition

$$\int\limits_0^\pi\int\limits_0^{2\pi}(|\Phi_1|^2+|\Phi_2|^2)\sin\vartheta d\vartheta d\varphi=1 \tag{19}$$

with using the relation (16) that yields $C=\sqrt{\frac{l+1}{\pi}}$. These spinors form an orthonormal basis in $L^2_2(\mathbb{S}^2)$. Finally, it should be noted that the given spinors can be expressed through the *monopole spherical harmonics* $Y^l_{mn}(\vartheta,\varphi)=P^l_{mn}(\cos\vartheta)e^{-im\varphi}$ which naturally arise when considering twisted TICs of complex scalar field [1–4] but we shall not need it here. In general, for physical applications the condition (19) seems to be quite enough rather than explicit form of $\Phi_{\pm\lambda}$.

4 Increase of Hawking radiation for spinor particles

As follows from the above, when quantizing twisted TICs of spinors we can take the set of spinors

$$\Psi_{\pm\lambda} = \frac{1}{\sqrt{2\pi\omega}} e^{i\omega t} r^{-1} \begin{pmatrix} F_1(r, \pm\lambda)\Phi_{\pm\lambda} \\ F_2(r, \pm\lambda)\sigma_1\Phi_{\pm\lambda} \end{pmatrix} \tag{20}$$

as a basis in $L_2^4(\mathbb{R}^2 \times \mathbb{S}^2)$ and realize the procedure of quantizing, as usual, by expanding in the modes (20)

$$\Psi = \sum_{\pm\lambda} \sum_{l=|n|}^{\infty} \sum_{m=-l}^{l+1} \int_{\mu_0}^{\infty} d\omega (a^-_{\omega nlm}\Psi_\lambda + b^+_{\omega nlm}\Psi_{-\lambda}) \,,$$

$$\overline{\Psi} = \sum_{\pm\lambda} \sum_{l=|n|}^{\infty} \sum_{m=-l}^{l+1} \int_{\mu_0}^{\infty} d\omega (a^+_{\omega nlm}\overline{\Psi}_\lambda + b^-_{\omega nlm}\overline{\Psi}_{-\lambda}) \,, \tag{21}$$

where $\overline{\Psi} = \gamma^0\Psi^\dagger$ is the adjont spinor and ($\dagger$) stands for hermitian conjugation. As a result, the operators $a^\pm_{\omega nlm}$, $b^\pm_{\omega nlm}$ of (21) should be interpreted as the creation and annihilation ones for spinor particle in the gravitational field of the black hole, in the field of monopole with Chern number n and in the external electric field of black hole. As to the functions $F_{1,2}(r)$ of (20) then in accordance with Eqs. (11) we can get the second order equations for them in the form

$$a\partial_r a\partial_r F_{1,2} + a\left[\sqrt{a}\partial_r\left(\frac{1}{2}\frac{d\sqrt{a}}{dr} \pm \frac{\lambda}{r}\right) + \frac{1}{4}\left(\frac{d\sqrt{a}}{dr}\right)^2 - \frac{\lambda^2}{r^2}\right]F_{1,2} =$$

$$\left[a\mu_0^2 - \left(\omega + \frac{eQ}{r}\right)^2\right]F_{1,2} \,. \tag{22}$$

By replacing

$$r^* = r + \frac{r_+^2}{r_+ - r_-}\ln\left|\frac{r - r_+}{2M}\right| - \frac{r_-^2}{r_+ - r_-}\ln\left|\frac{r - r_-}{2M}\right| \tag{23}$$

and by going to the dimensionless quantities $x = r^*/M, y = r/M, k = \omega M$ the equations (22) can be rewritten in the Schrödinger-like equation form

$$\frac{d^2}{dx^2}E_{1,2} + [k^2 - (\mu_0 M)^2]E_{1,2} = V_{1,2}(x, k, \alpha, \lambda)E_{1,2} \tag{24}$$

with $E_{1,2} = E_{1,2}(x, k, \alpha, \lambda) = F_\pm(Mx), F_\pm(r^*) = F_{1,2}[r(r^*)]$ while the potentials $V_{1,2}$ are given by

$$V_{1,2}(x, k, \alpha, \lambda) = \frac{1}{4}\frac{[y(x) - \alpha^2]^2}{y^6(x)} +$$

$$\left[-\frac{1}{2}\frac{3\alpha^2-2y(x)}{y^4(x)} \mp \frac{\lambda}{y^2(x)}\sqrt{1-\frac{2}{y(x)}+\frac{\alpha^2}{y^2(x)}+\frac{\lambda^2}{y^2(x)}}\right]\left[1-\frac{2}{y(x)}+\frac{\alpha^2}{y^2(x)}\right]+$$
$$\left[\frac{\alpha^2}{y^2(x)}-\frac{2}{y(x)}\right](\mu_0 M)^2-\frac{2keQ}{y(x)}-\frac{e^2Q^2}{y^2(x)}\,, \tag{25}$$

where $y(x)$ is a function reverse to the following one

$$x(y) = y+\frac{y_+^2}{y_+-y_-}\ln\left|\frac{y-y_+}{2}\right|-\frac{y_-^2}{y_+-y_-}\ln\left|\frac{y-y_-}{2}\right|, y_\pm = 1\pm\sqrt{1-\alpha^2}, \tag{26}$$

so $y(x)$ is the one-to-one correspondence between $(-\infty,\infty)$ and (y_+,∞).

Further we shall for the sake of simplicity restrict ourselves to the SW black hole case ($Q = 0$) and massless spinors ($\mu_0 = 0$). Then, as can be seen, when $x \to +\infty$, $V_{1,2} \to 0$ and at $x \to -\infty$, $V_{1,2} \to 1/64$. This allows us to pose the scattering problem on the whole x-axis for Eq. (24) at $k > 0$

$$E^+_{1,2}(x,k) \sim \begin{cases} e^{ikx} + \ s^{(1,2)}_{12}e^{-ikx} + \frac{1}{64k^2}, & x \to -\infty, \\ s^{(1,2)}_{11}e^{ikx}, & x \to +\infty, \end{cases}$$

$$E^-_{1,2}(x,k) \sim \begin{cases} s^{(1,2)}_{22}e^{-ikx} + \frac{1}{64k^2}, & x \to -\infty, \\ e^{-ikx} + s^{(1,2)}_{21}e^{ikx}, & x \to +\infty \end{cases} \tag{27}$$

with S-matrices $\{s^{(1,2)}_{ij} = s^{(1,2)}_{ij}(k,\lambda)\}$. Then by virtue of (11) one can obtain the equality

$$s^{(1)}_{11}(k,\lambda) = -s^{(2)}_{11}(k,\lambda)\,. \tag{28}$$

Having obtained these relations, one can speak about the Hawking radiation process for any TIC of spinor field on black holes. Actually, one can notice that Eq. (2) corresponds to the lagrangian

$$\mathcal{L} = \frac{i}{2}|g|^{1/2}\left[\overline{\Psi}\gamma^\mu\nabla_\mu\Psi - (\nabla_\mu\overline{\Psi})\gamma^\mu\Psi - \mu_0\overline{\Psi}\Psi\right]\,, \tag{29}$$

and one can use the energy-momentum tensor for TIC with the Chern number n conforming to the lagrangian (29)

$$T_{\mu\nu} = \frac{i}{4}\left[\overline{\Psi}\gamma_\mu\nabla_\nu\Psi + \overline{\Psi}\gamma_\nu\nabla_\mu\Psi - (\nabla_\mu\overline{\Psi})\gamma_\nu\Psi - (\nabla_\nu\overline{\Psi})\gamma_\mu\Psi\right]\,, \tag{30}$$

to get, according to the standard prescription (see, e. g., Ref. [11]) with employing (19) and (28), the luminosity $L(n)$ with respect to the Hawking radiation for TIC with the Chern number n (in usual units)

$$L(n) = \lim_{r\to\infty}\int\limits_{S^2} <0|T_{tr}|0> d\sigma = A\sum_{\pm\lambda}\sum_{l=|n|}^{\infty}2(l+1)\int\limits_0^\infty\frac{|s^{(1)}_{11}(k,\lambda)|^2}{e^{8\pi k}+1}dk \tag{31}$$

with the vacuum expectation value $< 0|T_{tr}|0 >$ and the surface element $d\sigma = r^2\sin\vartheta d\vartheta\wedge d\varphi$ while $A = \frac{c^5}{GM}\left(\frac{c\hbar}{G}\right)^{1/2} \approx 0.125728\cdot 10^{55}\,\mathrm{erg}\cdot\mathrm{s}^{-1}\cdot M^{-1}$ (M in g).

We can interpret $L(n)$ as an additional contribution to the Hawking radiation due to the additional spinor particles leaving black hole because of the interaction with monopoles and the conforming radiation should be called *the monopole Hawking radiation*. Under this situation, for the total luminosity L of black hole with respect to the Hawking radiation concerning the spinor field to be obtained, one should sum up over all n, i. e.

$$L = \sum_{n\in\mathbb{Z}} L(n) = L(0) + 2\sum_{n=1}^{\infty} L(n)\,, \tag{32}$$

since $L(-n) = L(n)$.

As a result, we can expect marked increase of Hawking radiation from black holes for spinor particles. But for to get an exact value of this increase one should apply numerical methods, so long as the scattering problem for general equation (24) does not admit any exact solution and is complicated enough for consideration — the potentials $V_{1,2}(x,k,\alpha,\lambda)$ of (25) are given in an implicit form. One can remark that, for instance, in the Schwarzschild black hole case the similar increase for complex scalar field can amount to 17 % of total (summed up over all the TICs) luminosity [2]. We hope to obtain numerical results elsewhere.

5 Concluding remarks

It is clear that the next important case is the Kerr black hole one or, more generally, the Kerr-Newman black hole one but the equations here will be more complicated. Besides, as was mentioned above, the corresponding scattering problems require serious study since contribution of TICs (e. g., to Hawking radiation) can be computed only numerically and when doing so it is very important to know whether the elements of the corresponding S-matrices exist in the strict mathematical sense which enables one to get exact, for example, integral equations for their numerical computation. It is dufficult task and it has still been studied only in a number of cases for complex scalar field [2,12].

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References

[1] Yu. P. Goncharov and J. V. Yarevskaya, *Mod. Phys. Lett.* **A 9** (1994) 3175; **A 10** (1995) 1813;

Yu. P. Goncharov, *Nucl. Phys.* **B 460** (1996) 167.

[2] Yu. P. Goncharov and N. E. Firsova, *Int. J. Mod. Phys.* **D 5** (1996) 419; *Nucl. Phys.* **B 486** (1997) 371.

[3] Yu. P. Goncharov, *Int. J. Mod. Phys.* **A 12** (1997) 3347; *Pis'ma v ZhETF* **67** (1998) 1021; *Mod. Phys. Lett.* **A 13** (1998) 1495.

[4] Yu. P. Goncharov, *Phys. Lett.* **B 398** (1997) 32; *Int. J. Mod. Phys.* **D 8** (1999) 123.

[5] *Géométrie Riemannian en Dimension 4. Seminaire Arthur Besse* (Cedic/Fernand Nathan, Paris, 1981).

[6] H. B. Lawson, Jr. and M.-L. Michelsohn, *Spin Geometry* (Princeton U. P., Princeton, 1989).

[7] R. Geroch, *J. Math. Phys.* **9** (1968) 1739;

C. J. Isham, *Proc. R. Soc. London* **A 364** (1978) 591.

[8] A. L. Besse, *Einstein Manifolds* (Springer-Verlag, Berlin, 1987).

[9] N. Ya. Vilenkin, *Special Functions and Theory of Group Representations* (Nauka, Moscow, 1991).

[10] M. Cahen and S. Gutt, *Simon Stevin* **62** (1988) 209;

M. Cahen, A. Franc and S. Gutt, *Lett. Math. Phys.* **18** (1989) 165.

[11] D. V. Gal'tsov, *Particles and Fields in the Vicinity of Black Holes* (Moscow University Press, Moscow, 1986).

[12] N. E. Firsova, *Int. J. Mod. Phys.* **D 7** (1998) 509.